Quantum Witnesses in the Presence of Depolarizing Noise

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Research Proposal: How does the presence of different levels of depolarizing noise impact the validity of the evaluations of quantum witnesses?

Abstract

To develop useful and powerful quantum computers, entanglement is a crucial tool to be able to understand and manipulate. In this paper, we aim to further this understanding by describing the use of quantum witnesses and comparing their performance in both ideal and noisy environments. An entanglement witness is a mathematical tool that can distinguish between an entangled state and a separable state. To test the witness, we prepared a simple Φ+ Bell state by applying an H gate to the first qubit then entangling them with a CNOT gate, designed a circuit for measurement across different axes, and used the witness to evaluate the correlation of the state with the equation $\langle W \rangle = 1 - X \otimes X - Y \otimes Y - Z \otimes Z$, where entanglement is true for all $\langle W \rangle < 0$. We then rerun the simulation at different levels of depolarizing noise. We found that the witness fails to consistently evaluate correctly at some probability p, where $0.15 < p < 0.25$. But, the witness's results are reasonably consistent both with and without noise, with the witness evaluation components each being within a few hundredths of a decimal of other simulations at the same level of noise. Therefore, quantum witnesses, while not the most well-suited to NISQ hardware, should be a useful method of measuring entanglement in future, noise-resistant systems.

Introduction

Entanglement is a key part of the advancement of quantum computing. It has numerous applications, including quantum cryptography, quantum teleportation, and notably, superdense coding. However, it remains a challenge to actually identify non-simulated quantum entanglement. Without the ability to perfectly entangle qubits, noise will persist in quantum computing, posing a significant hurdle for the field. Numerous methods have been developed to verify entanglement, such as Bell inequalities and spin squeezing inequalities. However, perhaps the most promising method is the use of quantum witnesses, as they enable entanglement detection without requiring complete knowledge of a quantum state's density matrix. Furthermore, they are fairly simple to implement into code in languages such as Google Cirq.

In this study, we will test the entanglement of the prepared Bell state by using quantum witnesses under various circumstances. To ensure accuracy, we will evaluate the performance of quantum witnesses across multiple noise levels and states. This provides a summary of the robustness of quantum witnesses, contributing to a broader assessment of their benefits and shortcomings. The use of viable quantum witnesses is of great importance, as they are a possible solution for determining the possibility of entanglement in multi-qubit (>2) systems, such as the famous GHZ states.

Methodology

To explore how noise affects the reliability of entanglement witnesses, we used **Cirq**, a quantum programming framework developed by Google, to simulate quantum circuits. Specifically, we generated a Bell state ($\Phi+$), a well-known example of a maximally entangled state, by applying a Hadamard gate to the first qubit, then entangling them with a CNOT gate. If

measuring along the X axis, we applied a Hadamard gate to each of the two qubits. If measuring along the Y axis, an Ry gate was used, and the state was rotated by $-\pi/2$.

We created two versions of the simulation:

- **Ideal (noiseless)**: Simulated the Bell state under perfect conditions, without any environmental noise.
- **Noisy**: Introduced realistic quantum noise using Cirq's built-in depolarizing noise models to mimic how actual quantum hardware behaves.

In both cases, we measured the correlation between the two qubits using specific measurements along the X, Y, and Z axes. These measurements help us determine whether or not the state is entangled. We then used an Entanglement Witness, an observable that evaluates whether or not a prepared state is entangled, to determine the entanglement of the Bell state in both the ideal and noisy simulations. This entanglement was identified by calculating the witness expectation value. The witness expectation value $\langle W \rangle$ is defined by subtracting one from the expectation values of each axis ($\langle W \rangle = 1 - \langle X \otimes X \rangle - \langle Y \otimes Y \rangle - \langle Z \otimes Z \rangle$). If the final witness expectation value is below 0, the witness has verified entanglement. But if it is above, the relationship is inconclusive.

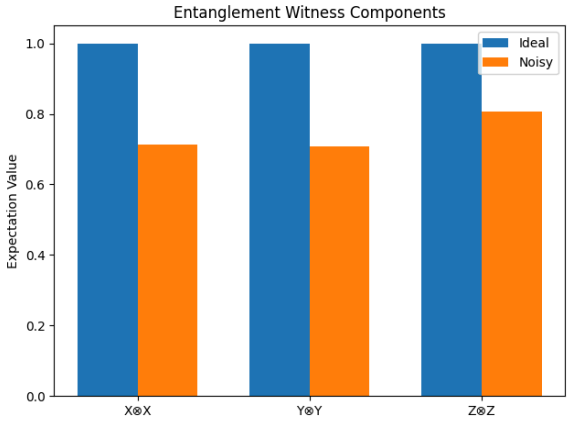
The simulation was run 1,000 times for each trial, and the average expectation value was plotted in a bar graph. The simulation was run twice for each iteration, once for an ideal state and once for a circuit with different levels of noise. The noise levels started at a probability $p = 0.05$ and ran for three trials, then increased by 0.10 up to a probability of 0.25. The model of noise applied was depolarization. Finally, we simulated the results using DensityMatrixSimulator and visualized them using a bar graph that compares the measurements under ideal and noisy

conditions. This allows us to see the extent to which noise interferes with the entanglement generation process.

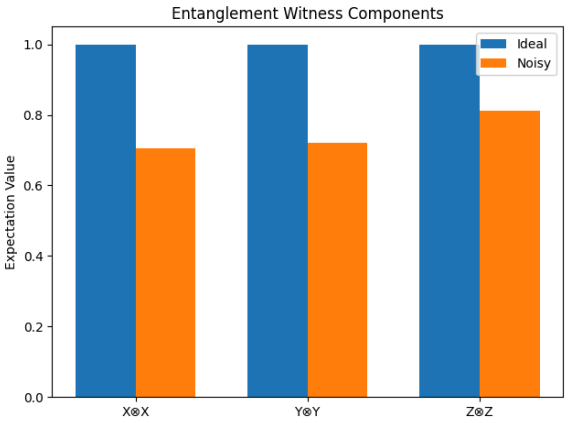
Findings

5% Noise

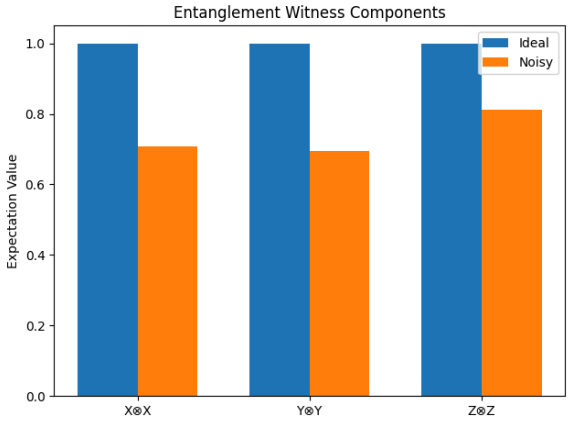
Trial 1



Trial 2

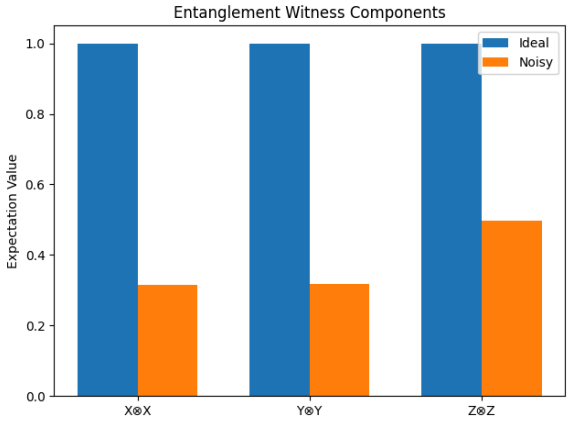


Trial 3

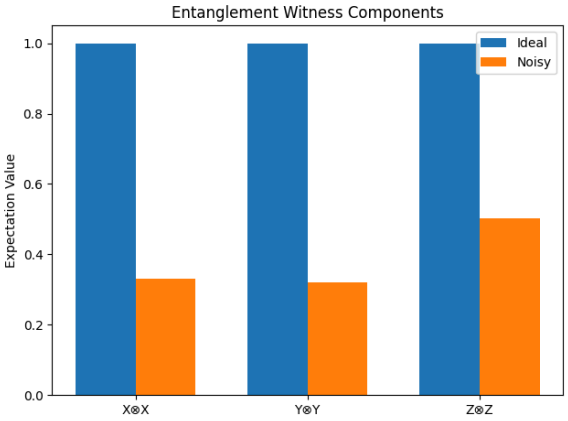


15% Noise

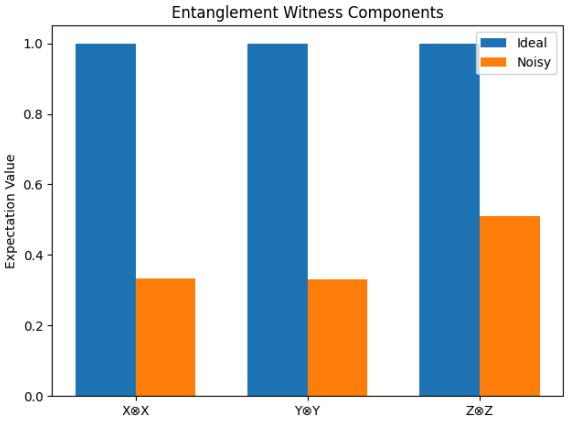
Trial 1



Trial 2

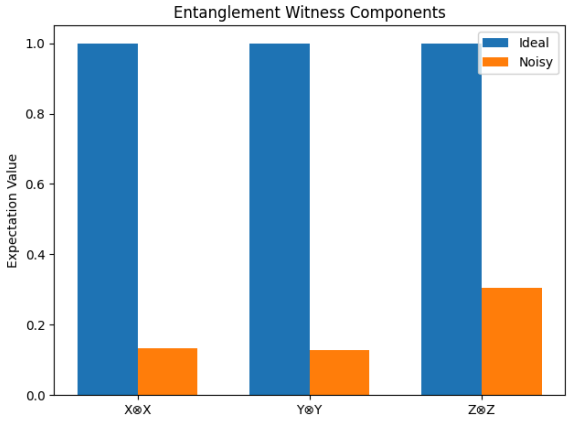


Trial 3

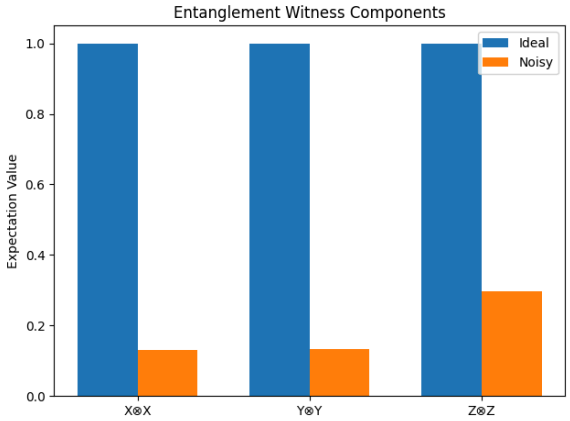


25% Noise

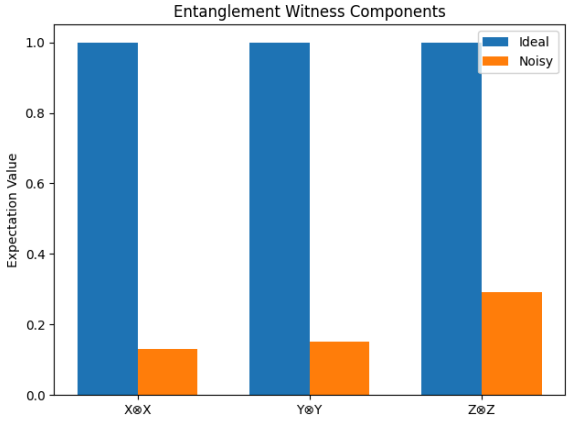
Trial 1



Trial 2



Trial 3



To evaluate the robustness of witnesses in the presence of noise, we initiated the simulation using a Bell State (Φ+) to represent a common entangled state. The entanglement witness expectation is calculated by $\langle W \rangle = 1 - X \otimes X - Y \otimes Y - Z \otimes Z$. If $\langle W \rangle < 0$, the prepared state is entangled, according to the witness. In ideal simulations, the witness components are each about one, demonstrating strong entanglement. In less noisy systems, such as the trials at five percent, there is a notable reduction in the expectation value, though the witness still correctly identifies the state as entangled. In systems with greater noise, the witness value is significantly reduced and produces a positive integer.

Throughout the graphs, there is a trend of the $Z \otimes Z$ expectation value being much greater than the $X \otimes X$ and $Y \otimes Y$ values. At 5% and 15% noise, the witness reliably evaluates the state to be entangled. The 5% model easily fulfills the >0 threshold, while the 15% model is less clear, but consistently under the proper mark. The 25% model is incorrect, though, as $X \otimes X$ and $Y \otimes Y$ values are about 0.1, and $Z \otimes Z \approx 0.3$. When computing the final witness expectation, we get consistent values of $\langle W \rangle \approx 1 - 0.1 - 0.1 - 0.3 \approx 0.5$, a positive decimal and thus a separable state.

Discussion

According to the simulation, the quantum witness protocol can accurately predict entanglement in ideal, noiseless environments. When observing the maximally entangled Bell State Φ+ in a perfect setting, the witness accurately computes an expectation value of ≈ -2, suggesting a heavily entangled state. However, at low levels of noise, the witness's capabilities are noticeably degraded, with each of the $\langle X \otimes X \rangle$, $\langle Y \otimes Y \rangle$, and $\langle Z \otimes Z \rangle$ components outputting significantly reduced values, as the 5% noise trials each equate to a final witness expectation value of ≈ -1.2, still entangled but impaired. The 15% trials demonstrate an even more noticeable

decline, as the final witness expectation for each trial is ≈ -0.1, just barely recognizing the entanglement. This exhibits the sensitivity of quantum witnesses to noise, particularly decoherence and loss of phase information. Another indication is that the Z⊗Z values are higher than the X⊗X and Y⊗Y values. Because the Φ+ state is naturally aligned with the Z axis, no additional gates need to be applied. Because of this, the integrity of the Z⊗Z component is noticeably higher than X⊗X and Y⊗Y, as there is less room for noise on the circuit, and thus a higher signal of entanglement. This demonstrates the effect that a mere one extra gate on each qubit can have due to noise.

At higher noise levels (such as the 25% trials), the witness expectation value is >0, meaning the witness believes it to be a non-entangled state. Therefore, this means that noisy systems either produced false negatives for the witness or caused the entangled system to become separable. Furthermore, this means that while witnesses are predictable and quite reliable for less noisy and ideal systems, they lack the robustness to be used accurately in systems exceeding moderate levels of noise, with our threshold being somewhere between 15% and 25% for reliably verifying Bell states. These results contrast with those of the previous experiment conducted by Prajapati et al. (2025), who found that when simulating noise, the witness vanished entirely beyond a probability of 3.7%. However, since their experiment was conducted on real quantum hardware, there is undoubtedly an unaccounted amount of non-simulated noise. Interestingly, they also found that the witness disappeared at a noise probability of 18.5%, but their results used Markovian amplitude damping, as opposed to our depolarizing noise model.

Since the witness has shown entanglement to be particularly sensitive to noise, witnesses may struggle in NISQ devices, where entanglement is very easily destroyed by noise.. However, as the graphs of this experiment demonstrate, witnesses are highly predictable and, when noise is minimized, accurate. Therefore, in the next generation of fault-tolerant systems, quantum witnesses appear to be a viable option for verifying entanglement. For a more robust experiment, a more complicated state would be a better testing method, as it would further push the abilities of the quantum witness. Also, testing on systems with many more entangled qubits in action could be a lot more indicative of the limitations of witnesses, though, from the performance of the witness in a two-qubit system, the results likely would not be promising.

Literature Review

Gühne and Tóth (2009) present a detailed analysis of entanglement detection methods, which serve as fundamental tools for quantum communication and encryption. The authors discuss theoretical detection methods, including Bell inequalities and entanglement witnesses and spin squeezing inequalities, together with experimental approaches that use repeated measurements on multiple quantum systems. The detection of entanglement remains challenging for large or noisy systems because it serves as a fundamental requirement for quantum key distribution, yet researchers agree on its necessity. Researchers agree that entanglement detection is vital for validating quantum system security and functionality, yet they disagree about which detection approaches offer the best combination of effectiveness and scalability. The current scientific understanding lacks a single detection method that works for all system types, especially when dealing with

high-dimensional or multiparty systems. Our quantum-secured email metadata project requires entanglement verification during key generation to establish encryption strength because this verification process is essential for our project. The simulation will benefit from entanglement witnesses and other verification techniques, which we can implement based on the findings presented in this literature, to protect against interception and key weaknesses.

Chruściński and Sarbicki demonstrate how entanglement witnesses enable the verification of entangled quantum states through state-independent methods. The authors analyze multiple witness types by demonstrating their construction from positive maps, Bell inequalities, and realignment tests. The paper illustrates how these witnesses enable the distinction between entangled and non-entangled states through geometric analysis of their structure. The main contribution of their research focuses on the distinction between decomposable witnesses, which function for particular entangled states, and indecomposable witnesse,s which detect various hidden entanglement types. The authors discuss different methods to construct witnesses, yet acknowledge that developing effective methods for complex and large systems remains challenging. The research demonstrates how entanglement witnesses could serve as a tool to verify quantum key security. The tools would enable us to determine if the key remains entangled following potential attacks , which is essential for maintaining strong metadata encryption in our simulation.

Prajapati, Harsh, et al. (2025) provide a helpful framework for our experiment, which will take place on a real quantum processor. In the study, they analytically prepared an entanglement witness to test the entanglement of four-qubit Dicke states. They tested the witness in the noisy quantum processor and with additional different types of simulated noise. They

found the witness to be very sensitive to increasing levels of noise. Their study provides a helpful comparison and data points to inform the creation of a tested hypothesis for our results.

Bas Dirkse et al. provide a more mathematically rigorous definition of a quantum witness that remains easy to understand. It helped inform us of the mathematical theory of entanglement witnesses. In their study, they also developed an easily reproducible method for detecting entanglement without assuming whether the underlying noise is identical or identically distributed. However, their findings are outside the scope of our experiment.

Conclusion

This research highlights the significant impact that environmental noise has on the effectiveness of quantum witnesses in detecting entangled states. Through simulations of both ideal and noisy quantum systems, we demonstrated that quantum witnesses can reliably identify entanglement at low noise levels but become increasingly less reliable as noise increases. Our results show a consistent decrease in witness expectation values as noise rises from 5% to 25%, with higher noise rates causing entangled states to be mistakenly identified as separable. These findings demonstrate the importance of accounting for realistic noise conditions when utilizing more robust witnesses or noise-resilient methods, which will be crucial for advancing quantum technologies, particularly in applications such as quantum cryptography and secure communication.

References

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